

Unit-1 Binary Systems

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Digital System

- It is a system that manipulates (processes) discrete elements of information.
- A digital system is a discrete information processing system.
- **For e.g.** calculator, digital voltmeters, general purpose computer etc.
- In such systems, all quantities are represented using two values i.e. 0 and 1.

Analog System

- It is a system that manipulate physical quantities that are represented in analog form i.e. continuous range of values.
- **For e.g.** voltmeter, speedometer, analog thermometer etc.

Block Diagram of Digital Computer

A digital computer is a programmable machine which read the binary instruction and processes the data which are presented in binary form.

The digital computer takes the binary data at input, processes according to the set of instructions called program and produces the digital output.

Fig: Block diagram of a Digital Computer

Working principles of digital computer:

- 1. Memory unit stores programs as well as input, output and intermediate data.
- 2. The control unit supervises the flow of information between various units and retrieve the instructions stored in memory unit.
- 3. After getting control signal memory unit sends the data to the processor.
- 4. For each instruction control unit informs the processor to execute the operation according to the instruction.
- 5. After getting control signal processor sends the process information to memory unit and memory unit sends those information to the output unit.

Advantages of digital system

- In case of digital system large number of ICs are available for performing various operations hence digital systems are highly reliable, accurate, small in size and speed of operation is very high.
- Computer controls digital system can be controlled by software that allows new function to be added without changing hardware.
- Less expensive
- Easy to manipulate

Disadvantages of digital system

- It is difficult to install digital system because it required many more complex electronic circuits and ICs.
- In digital systems, if a single piece of data lost, large blocks of related data can completely change.

Number System

In general, in **any number system there is an ordered set of symbols known as digits** with rules defined for performing arithmetic operations like addition, multiplication etc. A collection of these digits makes a number in general has two parts- integer and fractional. Set apart by a radix point (.), i.e.

 $N = a$ number and $r =$ radix or base of number system

In general, a number expressed in base-r system has coefficients multiplied by powers of r:

$$
A_{n-1}r^{n-1} + A_{n-2}r^{n-2} + \dots + A_1r^1 + A_0r^0 + A_{-1}r^{-1} + A_{-2}r^{-2} + \dots + A_{-m}r^{-m}
$$

For example, $(3456.54)_{10}$ can be written as:

 $3 \times 10^3 + 4 \times 10^2 + 5 \times 10^1 + 6 \times 10^0 + 5 \times 10^{-1} + 4 \times 10^{-2}$

 \triangleright There are mainly four number system which are used in digital electronics platform.

1. Decimal number system:

- The decimal number system contains ten unique digits from 0 to 9.
- The base or radix is 10.
- *2. Binary number system:*
	- The binary number system contains two unique digits 0 and 1.
	- The base or radix is 2.
- *3. Octal number system:*
	- The octal number system contains eight unique digits from 0 to 7.
	- The base or radix is 8.
- *4. Hexadecimal number system:*
	- The hexadecimal number system contains sixteen unique digits: 0 to 9 and six letters A, B, C, D, E and F.
	- The base or radix is 16.

Number Base Conversion

The possibilities:

Decimal to Other Base System

The decimal number can be an integer or floating-point integer. When the decimal number is a floating-point integer, then we convert both part (integer and fractional) of the decimal number in the isolated form (individually). There are the following steps that are used to convert the decimal number into a similar number of any base $'r'$.

Conversion of Integer part:

- Divide the given integer part of decimal number by base $'r'$ successively and write down all the remainders till the quotient is zero.
- Write all the remainders starting with the MSB (Most Significant Bit) i.e. from bottom to LSB (Least Significant Bit) i.e. top.

Conversion of Fractional part:

- Multiply the given fractional part of decimal number by base $'r'$ successively until the fractional part becomes zero.
- Note down the integer part starting from first. *Note:* If fractional part does not become zero then, result has been taken up to 6 places.

Decimal to Binary Conversion

1. Convert $(51)_{10}$ *and* $(152)_{10}$ *into binary.*

$$
\frac{2}{2}\frac{51}{25}
$$

\n
$$
\frac{2}{2}\frac{12}{12}
$$

\n
$$
\frac{2}{2}\frac{1}{38}
$$

\n
$$
\frac{2}{2}\frac{1}{9}
$$

\n
$$
\frac{2}{2}\frac{1}{4}
$$

\n
$$
\frac{2}{2}\frac{1}{2}
$$

\n
$$
\frac{2}{2}\frac{1}{
$$

2 152 252

2. *Convert* (41.6875)₁₀ into binary.

Conversion of integer part: Conversion of fractional part:

$$
\begin{array}{c|cc}\n2 & 41 \\
\hline\n2 & 20 \\
\hline\n2 & 10 \\
\hline\n2 & 5 \\
\hline\n2 & 2 \\
\hline\n2 & 1 \\
\hline\n0 & 1 (MSB)\n\end{array}
$$

$$
(41)_{10} = (101001)_2
$$

$$
\therefore (41.6875)_{10} = (101001.1011)_2
$$

 $(0.6875)_{10} = (0.1011)_2$

Decimal to Octal Conversion

1. Convert $(125)_{10}$ *into octal.*

$$
(125)_{10} = (175)_8
$$

2. *Convert* (153.513)₁₀ into octal.

Conversion of integer part: Conversion of fractional part:

$$
(153)_{10} = (231)_8
$$

∴ $(153.513)_{10} = (231.406517)_8$

	integer	fraction
0.513×8	4	0.104
0.104×8	0	0.832
0.832×8	6	0.656
0.656×8	5	0.248
0.248×8		0.984
0.948×8		0.584

$$
(0.513)_{10} = (0.406517)_{8}
$$

3. *Convert* (125.6875)₁₀ to octal.

8 125 8 15 8 1 0 7 1 (MSB)

$$
(125)_{10} = (175)_8
$$

∴ $(125.6875)_{10} = (175.54)_{8}$

Conversion of integer part: Conversion of fractional part:

		integer \vert fraction
5 (LSB)	0.6875×8 5	0.5000
	0.5000×8	0.0000

$$
(0.6875)_{10} = (54)_8
$$

Decimal to Hexadecimal Conversion

1. Convert $(2598)_{10}$ *to hexadecimal.*

$$
\begin{array}{c|cc}\n16 & 2598 \\
\hline\n16 & 162 \\
\hline\n16 & 10 \\
\hline\n0 & & 10(A) (MSB)\n\end{array}
$$

$$
(2598)_{10} = (A26)_{16}
$$

2. *Convert* $(952.62)_{10}$ *to hexadecimal.*

$$
(952)_{10} = (3B8)_{16}
$$

Conversion of integer part: Conversion of fractional part:

$$
(0.513)_{10} = (0.9E B 851)_{16}
$$

∴ $(952.62)_{10} = (3B8.9EB851)_{16}$

Any Base to Decimal Conversion

Converting from any base to decimal is done by multiplying each digit by its corresponding positional weights and summing.

Binary to Decimal

1. Convert $(10110)_2$ *into decimal.*

 $(10110)₂ = 1 \times 2⁴ + 0 \times 2³ + 1 \times 2² + 1 \times 2¹ + 0 \times 2⁰$ $= 16 + 0 + 4 + 2 + 0$ $= 22$ ∴ $(10110)_2 = (22)_{10}$

2. *Convert* (1101.011)₂ into decimal.

 $(1101.011)₂ = 1 \times 2³ + 1 \times 2² + 0 \times 2¹ + 1 \times 2⁰ + 0 \times 2⁻¹ + 1 \times 2⁻² + 1 \times 2⁻³$ $= 8 + 4 + 0 + 1 + 0 + 0.25 + 0.125$ $= 13.375$ ∴ $(1101.011)₂ = (13.375)₁₀$

Octal to Decimal

1. Convert (724.25)₈ *into decimal.*

$$
(724.25)8 = 7 \times 82 + 2 \times 81 + 4 \times 80 + 2 \times 8-1 + 5 \times 8-2
$$

= 448 + 18 + 4 + 0.25 + 0.0781
= 470.3281

$$
\therefore (724.25)8 = (470.3281)_{10}
$$

2. *Convert* $(6301)_8$ *into decimal.*

$$
(6301)8 = 6 \times 83 + 3 \times 82 + 0 \times 81 + 1 \times 80
$$

= 3072 + 192 + 0 + 1
= 3265

$$
\therefore (6301)8 = (3265)10
$$

Hexadecimal to Decimal

1. Convert $(AOF9.0EB)_{16}$ *to decimal.*

 $(A0F9.0EB)_{16}$ $= 10 \times 16^3 + 0 \times 16^2 + 15 \times 16^1 + 9 \times 16^0 + 0 \times 16^{-1} + 14$ \times 16⁻² + 11 \times 16⁻³ $= 40960 + 0 + 240 + 9 + 0 + 0.0546 + 0.0026$ $= 41209.0572$

∴ $(A0F9.0EB)_{16} = (41209.0572)_{10}$

2. *Convert* $(A9F5. DE)_{16}$ *to decimal.*

Octal and Hexadecimal number to Binary

Octal to Binary

To convert Octal number to its Binary equivalent, each digit of given octal number is directly converted to its 3-bit binary equivalent.

Q. Convert (367.52)₈ *into Binary.*

$$
(367.52)8 = 011 110 111.101 010
$$

= 11110111.10101

 \therefore (367.52)₈ = (11110111.10101)₂

Hexadecimal to Binary

To convert Hexadecimal number to its Binary equivalent, each digit of given hexadecimal number is converted to its 4-bit binary equivalent.

Q. Convert (3A9E. **B0D**)₁₆ into Binary.

 $(3A9E. B0D)₁₆ = 0011 1010 1001 1110.1011 0000 1101$ = 11101010011110.101100001101

∴ $(3A9E.B0D)₁₆ = (11101010011110.101100001101)₂$

Binary to Octal and Hexadecimal Numbers

Binary to Octal

As $8 = 2³$, for binary to octal conversion groups of 3 binary bits each are formed in the binary number. After forming groups, each group of three binary bits is converted to its octal equivalent.

- For integer part of the binary number, the group of three bits is formed from right to left. In the binary fraction the group of three bits is formed from left to right. If there are not 3 bits available at last, just stuff '0' to make 3 bits group.

l. Convert (10110001101011.111100000110)₂ into octal.

 $(10110001101011.111100000110)_2 = 010 110 001 101 011.111 100 000 110$ $= 2$ 6 1 5 3 7 4 0 6 $= 26153.7406$

∴ (10110001101011.111100000110)₂ = (26153.7406)₈

2. *Convert* $(110101.101010)_2$ *into octal.*

 $(110101.101010)₂ = 110101.101010$ $= 65.52$

∴ $(110101.101010)_2 = (65.52)_8$

Binary to Hexadecimal

As $16 = 2⁴$, for binary to hexadecimal conversion groups of 4 binary bits each are formed in the binary number. After forming groups, each group of four binary bits is converted to its hexadecimal equivalent.

- For integer part of the binary number, the group of four bits is formed from right to left. In the binary fraction the group of four bits is formed from left to right. If there are not 4 bits available at last, just stuff '0' to make 4 bits group.

1. Convert (10110001101011.111100000110)₂ into hexadecimal.

 $(10110001101011.111100000110)_2 = 0010 1100 0110 1011.1111 0000 0110$
= 2 C 6 B F 0 6 $= 2$ C 6 B F 0 6 $= 2C6B.F06$

 \therefore (10110001101011.111100000110)₂ = (2C6B.F06)₁₆

Conversion of Octal and Hexadecimal numbers

Octal to Hexadecimal

Steps to convert from octal to its hexadecimal equivalent:

- Each digit of given octal number is converted into its 3-bit binary equivalent.
- Now, form the groups of 4 binary bits to obtain its hexadecimal equivalent.

Q. Convert (673.124) ₈ *into hexadecimal.*

 $(673.124)_{8} = (110 111 011 . 001 010 100)_{2}$ $= (0001 1011 1011 . 0010 1010)_2$ $= (1BB.2A)_{16}$

Hexadecimal to Octal

Steps to convert from hexadecimal to its octal equivalent:

- Each digit of given hexadecimal number is converted into its 4-bit binary equivalent.
- Now, form the groups of 3 binary bits to obtain its octal equivalent.

E.g.

Q. Convert (B9F.AE)₁₆ into octal.

 $(B9F.AE)_{16} = (1011\ 1001\ 1111\ 1010\ 1110)_2$ $=(101 110 011 111. 101 011 100)_2$ $= (5637.534)_{8}$

Complements

Complements are used in digital computers for simplifying the subtraction operation and for logical manipulations.

There are two types of complements for each base $-r$ system:

- a) The $r's$ complement and
- b) The $(r 1)$'s complement.
- \checkmark r's complement is known as 10's complement in base 10 and 2's complement in base 2.
- \checkmark ($r-1$)'s complement is known as 9's complement in base 10 and 1's complement in base 2.

\triangleright r's complement

Given a positive number N in base r with an integer part of n digits, the r 's complement of N is defined as

The r's complement of $N = \begin{cases} r^n - N, & \text{if } N \neq 0 \\ 0 & \text{if } N = 0 \end{cases}$ 0, if $N = 0$

E.g.

10's complement of $52520 = 10^5 - 52520 = 47480$ 10's complement of $0.3267 = 10^{0}$ -0.3267 = 0.6733 10's complement of $25.639 = 10^2 - 25.639 = 74.361$ 2's complement of $(101100)_2 = (2^6)_{10}$ - $(101100)_2 = (1000000 - 101100)_2 = 010100$ 2's complement of $(0.0110)₂ = (2⁰)₁₀$ -0.0110 = 0.1010

\triangleright $(r-1)'s$ complement

Given a positive number N in base r with an integer part of n digits and a fractional part of *m* digits, the $(r - 1)'s$ complement of *N* is defined as:

The $(r-1)$'s complement of $N = r^n - r^{-m} - N$

E.g.

9's complement of $(52520)_{10} = (10^5 \text{-} 10^0 \text{-} 52520) = 47479$

9's complement of $(0.3267)_{10}$ is $(10^0 - 10^{-4} - 0.3267) = 0.6732$

9's complement of $(25.693)_{10}$ is $(10^2 - 10^{-3} - 25.693) = 74.306$

1's complement of $(101100)_2$ is $(2^6-2^0)_{10}$ - $(101100)_2 = 111111$ -101100 = 010011

1's complement of (0.0110) ₂ is $(1-2⁴)_{10}$ - (0.0110) ₂ = 0.1001

Subtraction with Complements

Subtraction with r's Complements

Subtraction of two positive numbers $(M - N)$, both of base r, may be done as follows: Step-1: Add the minuend M to the $r's$ complement of subtrahend N .

Step-2: Inspect the result obtained in step 1 for an end carry:

(a) If an end carry occurs, discard it.

(b) If an end carry does not occur, take the r's complement of the number obtained in step 1 and place a negative sign in front.

E.g.

1. Using 10's complement, subtract (72532-3250).

Let, $M = 72532$ and $N = 03250$ 10's complement of $N = (10^5 - 03250) = 96750$ Now, 72532 +96750 1 69282 Here, end carry occurred. So discard it. So, answer = 69282

2. *Using 10's complement, subtract (3250-72532).*

Let, $M = 03250$ and $N = 72532$ 10's complement of $N = (10^5 - 72532) = 27468$ Now, 03250 +27468 30718 Here, no end carry. .

So, answer = $-(10)$'s complement of 30718) = $-(10^5 - 30718) = -69282$

3. Using 2's complement, subtract (1000100-1010100).

Let, $M = 1000100$ and $N = 1010100$ 2's complement of $N = (2^7)_{10} - (1010100)_2 = 10000000 - 1010100 = 0101100$ Now,

$$
1000100\n\n+0101100\n\n1110000
$$

No end carry. . So, answer = $-(2$'s complement of 1110000) = $-[(2^7)_{10} - (1110000)_2] = -10000$

4. *Subtract (1110.111-1010.101) using 2's complement.*

Let, $M = 1110.111$ and $N = 1010.101$ 2's complement of $N = (2^4)_{10} - (1010.101)_2 = 10000 - 1010.101 = 0101.011$ Now, 1110.111

$$
\frac{1110.111}{+0101.011}
$$

1 0100.010

End carry occurred. So discard it. So, answer = 0100.010

Subtraction with (r-1)'s Complements

The subtraction of $M - N$, both positive number in base r, may be calculated in the following manner:

Step-1: Add the minuend M to the $(r - 1)$'s complement of the subtrahend N. Step-2: Inspect the result obtained in step 1 for an end carry.

(a) If an end carry occurs, add 1 to the list significant digit (end-round carry)

(b) If an end carry does not occurs, take the $(r - 1)$'s complement of the number obtained in step 1 and place a negative sign in front.

E.g.

1. Using 9's complement, subtract (453.35-321.17).

Let, $M = 453.35$ and $N = 321.17$ 9's complement of $N = 10^3 - 10^{-2} - 321.17 = 678.82$ Now, 453.35

∴ Answer = 132.18

2. Using 9's complement, subtract (3250 – 72532).

Let, $M = 03250$ and $N = 72532$ 9's complement of $N = 10^5 - 10^0 - 72532 = 27467$ Now, 03250 +27467

Here, no end carry.

\nSo, answer =
$$
-(9
$$
's complement of 30717) = $-(10^5 - 10^0 - 30718) = -69282$

+0101011

3. Using 1's complement, subtract (1000100-1010100).

Let, $M = 1000100$ and $N = 1010100$ 1's complement of $N = 0101011$ Now, 1000100

1101111 No end carry. . So, answer = $-(1)$'s complement of 1101111) = -10000

4. Subtract (1010100 - 1000100) using 1's complement.

Let, $M = 1010100$ and $N = 1000100$ 1's complement of $N = 0111011$ 1010100 +0111011 0001111 $+1$ 0010000 End carry occurred. End round carry. ∴ Answer = 10000

Binary Codes

When numbers, letters or words are represented by a special group of binary symbols/combinations, we say that they are being encoded and the group of symbols is called a code. Some familiar binary codes are: Decimal Codes, Error-detection Codes, The Reflected Code, Alphanumeric Codes etc.

Decimal Codes

The representation of decimal digits by binary combinations is known as decimal codes. Binary codes from decimal digits require minimum of four bits. Numerous different codes can be obtained by arranging four or more bits in ten distinct possible combinations. Some decimal codes are-

- BCD
- Excess-3

Binary-Coded-Decimal (BCD) Code

If each digit of a decimal number is represented by its binary equivalent, the result is a code called binary coded decimal (BCD). It is possible to assign weights to the binary bits according to their positions. The weights in the BCD code are 8,4,2,1.

Excess-3 Code

This is an unweighted code. Its code assignment is obtained from the corresponding value of BCD after the addition of 3.

Error-detection Codes

- An error detection codes can be used to detect errors during transmission. A parity bit is an extra bit included with a message to make the total number of 1's either odd or even.
- For a message of four bits parity (P) is chosen so that the sum of all 1's is odd (in all five bits) or the sum of all 1's is even. In the receiving end, all the incoming bits (in this case five) are applied to a "parity-check" network for checking.
- An error is detected if the check parity does not correspond to the adopted one. The parity method detects the presence of one, three or any odd combination of errors. An even combination of errors is undetectable. Additional error-detection schemes may be needed to take care of an even combination of errors.

The Reflected Code/Grey Code

- The Reflected code, also called Gray code is unweighted and is not an arithmetic code; that is, there are no specific weights assigned to the bit positions.
- It is a binary numeral system where two successive values differ in only one bit (binary digit).
- For instance, in going from decimal 3 to decimal 4, the Gray code changes from 0010 to 0110, while the binary code changes from 0011 to 0100, a change of three bits. The only bit change is in the third bit from the right in the Gray code; the others remain the same.

Alphanumeric Code

- In order to communicate, we need not only numbers, but also letters and other symbols. In the strictest sense, alphanumeric codes are codes that represent numbers and alphabetic characters (letters). Most such codes, however, also represent other characters such as symbols and various instructions necessary for conveying information.
- The ASCII is the most common alphanumeric code.

ASCII Code

ASCII is the abbreviation for **American Standard Code for Information Interchange**. ASCII is a universally accepted alphanumeric code used in most computers and other electronic equipment. Most computer keyboards are standardized with the ASCII. When we enter a letter, a number, or control command, the corresponding ASCII code goes into the computer.

- ASCII has 128 characters and symbols represented by a 7-bit binary code. Actually, ASCII can be considered an 8-bit code with the MSB always 0. This 8-bit code is 00 through 7F in hexadecimal.
- The first thirty-two ASCII characters are non-graphic commands that are never printed or displayed and are used only for control purposes. Examples of the control characters are ""null," "line feed," "start of text," and "escape."
- The other characters are graphic symbols that can be printed or displayed and include the letters of the alphabet (lowercase and uppercase), the ten decimal digits, punctuation signs and other commonly used symbols.

Extended ASCII characters

In addition to the 128 standard ASCII characters, there are an additional 128 characters that were adopted by IBM for use in their PCs (personal computers). Because of the popularity of the PC, these particular extended ASCII characters are also used in applications other than PCs and have become essentially an unofficial standard. The extended ASCII characters are represented by an 8-bit code series from hexadecimal 80 to hexadecimal FF.

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Reference:

- *M. Morris Mano, "Digital Logic & Computer Design"*